Engineering Notes

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Potential Flow Past Axisymmetric Bodies at Angle of Attack

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Nomenclature

a = singularity line inset distance for blunt nosed body
C_p = pressure coefficient
L = body length
N = total number of axial singularity elements
R = maximum body radius
r = radial coordinate
SR = slenderness ratio
x = axial coordinate
θ = azimuthal coordinate

Introduction

USE of singularity methods based on Green's theorem has become a common technique for solving incompressible potential flow problems, due to its computational efficiency. Further, there has been a recent trend toward use of higher order singularities in the form of linearly and quadratically varying distributions. The purpose of the present Note is to document the accuracy of higher-order axial singularity techniques for the calculation of flow at angle of attack past an axisymmetric body.

von Karman[1] first used a piecewise constant line source singularity distribution for the calculation of axial flow past an axisymmetric body. Oberkampf and Watson[2] discussed limitations of the constant source singularity distribution due to von Karman, and found it necessary to use double precision arithmetic. Karamcheti[3] discussed von Karman's method and suggested methods of extending the technique for inclined flow by introducing a distribution of doublets with axes aligned with the cross flow direction. Zedan and Dalton[4-6] describe extensions of the method to include higher order (linear and quadratic) line source distributions, but they did not extend the method for nonzero angle of attack. Zedan and Dalton obtained highly accurate pressure coefficient distributions in axial flow using as few as 10 to 20 higher order source elements to represent various smooth body geometries. Also, they found that inset of the ends of the line of singularities from a blunt nose or tail was essential to the generation of accurate results, and they recommended using an inset distance equal to one percent of the body length.

Received March 15, 1983; revision received July 8, 1983.

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In the present work, Karamcheti's[3] suggestion to use higher order line singularity techniques has been developed to allow accurate calculation of incompressible flow past an axisymmetric body at angle of attack. Either piecewise-constant or linearly-varying line source or line doublet singularities have been utilized for the axial flow, while either constant or linear doublets, whose axes are aligned with the crossflow, have been utilized for the crossflow solution. For flows at angle of attack, the two velocity distributions are superimposed to obtain the resulting three dimensional surface velocity and pressure distributions. Details of the derivation of the method have been given in Refs. 7 and 8.

This Note presents results of a convergence study using this axial singularity method, where solution accuracy has been investigated for ellipsoids of slenderness ratios between two and ten, both for axial and inclined flow. Effects of singularity type, element number, element size distribution, and singularity line inset distance have been investigated. Further, a paneling scheme has been developed which yields accurate results for the class of axisymmetric bodies having continuous body slopes, but having discontinuous jumps in curvature, such as occurs at the juncture between an ellipsoidal or ogival nose and cylindrical body.

Results

The effect of inset of the ends of the singularity line away from the nose and tail of a blunt nosed body is demonstrated in Fig. 1 for axial flow past an $SR = 5$ ellipsoid. The root mean square error in pressure coefficient has been computed using the differences between the present numerical solution and the exact solution[9] evaluated at 40 points on the surface of the ellipsoid, at x coordinates equal to $x_i = 0.0125 + 0.025\ (i-1)$. These same surface points have been used throughout the present work to compare the accuracy of various singularity types and element distributions. In Fig. 1, a total of 20 equally-spaced piecewise constant source singularity elements have been used. The inset distance, a, has been varied from 0 to 2% of the body length. A minimum pressure coefficient error is observed for $a = 0.01L$ for this $SR = 5$ ellipsoid. Similar accuracies were obtained in Ref. 7 for inclined flow past the same $SR = 5$ ellipsoid using the same inset. However, it has been found that the optimal inset distance depends upon SR. At $SR = 10$, for example, best accuracy was obtained at $a = 0.0025L$, while for $SR = 2$, $a = 0.06L$ yielded the most accurate results both for zero and non zero angle of attack. This led to the discovery that the optimal inset distance depended upon the ratio of the nose radius of curvature to the body length, so that for ellipsoids the optimal inset was given by $a = (0.25/SR^2)L$.[7] Solution accuracy obtained using this optimum inset criterion was essentially independent of SR for $2 \leq SR \leq 10$ and for $0 \leq \alpha \leq 30$ deg.

The effects of singularity type, number of singularity elements, and the effects of cosine vs equal spacing of the elements has been investigated at $\alpha = 0$ deg, using the optimum inset for an $SR = 5$ ellipsoid. This is summarized in Fig. 2. The rate of convergence to the exact solution with increasing N varies between the different singularity types and orders. For a fixed number of elements, a cosine-type distribution of the elements having smaller elements near the nose and tail of the body yields nearly an order of magnitude decrease in error. Again, for a fixed number of elements, the

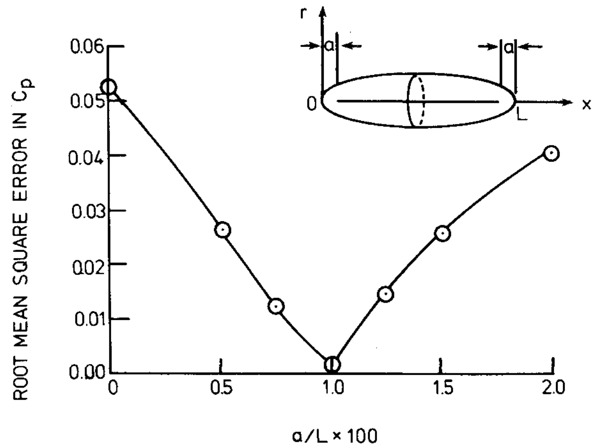

Fig. 1 Effect of inset of axial singularity distribution from nose and tail of $SR=5$ ellipsoid, $N=20$ equally-spaced constant source elements.

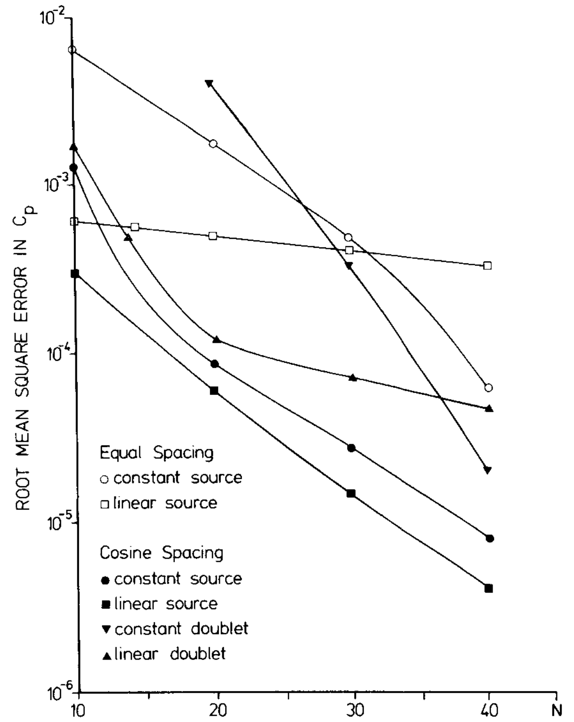

Fig. 2 Variation of RMS error in surface C_p with number of singularity elements, N, for an $SR=5$ ellipsoid at $\alpha=0$ deg, a = 0.01L.

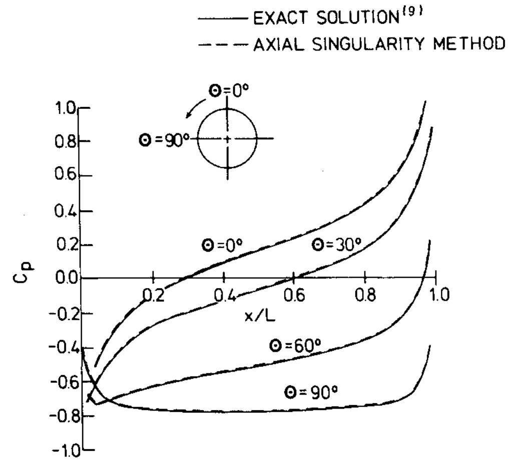

Fig. 3 Comparison between surface C_p distribution from axial singularity method and exact solution for an $SR=5$ ellipsoid at $\alpha=30$ deg. Results obtained using $N=20$ cosine spaced linear source elements and $N=20$ cosine spaced linear crossflow doublet elements with a = 0.01L.

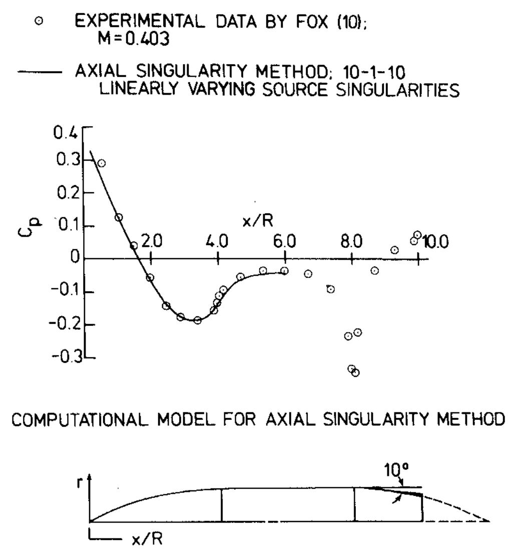

Fig. 4 Pressure coefficient distribution along meridian line for axial flow past a cylindrical body with ogival nose, using singularity method and compared with experimental data.

linear source singularity using cosing spacing is most accurate, followed by the constant source with cosine spacing. These two methods are consistently more accurate than the axial doublet singularities and yield C_p errors on the order of 10^{-4} for $N=20$. Since these errors are dominated by errors near the nose and tail, it has been found[7] that solution accuracy reaches five significant figures over the center 80% of the body length. Similar error magnitudes have been found for inclined flows for α values up to 30 deg. As an example, Fig. 3 compares calculated C_p distributions along various meridian lines with the exact solution[9] for an $SR=5$ ellipsoid at $\alpha=30$ deg. In this case, a total of 20 linear source elements and 20 linear doublet elements were used, both having a cosine spacing and optimal inset. As in the axial flow case, error was found to be on the order of 10^{-4} along all meridian lines.

Finally, Fig. 4 presents a comparison of results calculated using the axial singularity method of Refs. 7 and 8 with experimental data[10] for a cylindrical body having an ogival nose. This body has a continuous slope distribution along the body surface, but has a discontinuous change from finite to zero curvature at the nose-cylinder juncture. This type of axisymmetric body is one which previous authors[3-6] have claimed cannot accurately be modeled using axial singularity methods. However, as displayed in Fig. 4, axial singularity methods can yield accurate pressure coefficient results for such bodies, including inclined flow cases[7]. Note that the actual body in Ref. 10 had a truncated conical tail which cannot be modeled by the present theory. Similar results were presented in Ref. 7 for a cylindrical body with an ellipsoidal nose. As detailed in Ref. 7, successful calculation of such flows was only obtained by using the linear source singularity, deleting the source strength continuity requirement at the juncture where the discontinuity in curvature occurs, and replacing this equation with an additional control point on the cylinder surface. Further, for bodies having a relatively long cylindrical portion and a relatively short nose and tail, it was necessary to utilize three linear source singularity elements to represent the cylinder[7].

Discussion

The axial singularity methods developed in Refs. 7 and 8 have been shown to yield accurate pressure coefficient solutions with little computational expense for both axial and inclined flows. Use of the linear source singularity and linear crossflow doublet singularity with twenty cosine spaced elements and the optimal inset of the singularity line yielded pressure coefficient results which were accurate to four significant figures for ellipsoidal bodies of slenderness ratios between two and ten, over the entire 0-30 deg range of angle of attack studied. However, a key requirement for obtaining such solution accuracies was use of optimal inset of the singularity line away from the blunt nose and tail. For ellipsoidal bodies this optimal inset distance has been found to be proportional to the nose radius of curvature. No inset was utilized for sharp nosed bodies such as an ogival body. It is anticipated that the optimal inset should vary for other classes of axisymmetric bodies, but it is recommended that inset distances obtained using the criterion developed for ellipsoids be used initially for all blunt nosed bodies.

The axial singularity method has been extended to allow accurate pressure coefficient solutions to be obtained for axisymmetric bodies having discontinuous changes in surface curvature. However, axial singularity methods have not been found suitable for bodies having discontinuities in surface slope, such as a cone-cylinder. Surface singularity methods appear necessary for such geometries.Accurate solutions for bodies having jumps in curvature were obtained by using higher order source singularity distributions, but only by allowing discontinuous changes in source strength across the point where the jump in curvature occured. Indeed, the resulting source intensity distributions were observed to jump from a small positive value to a negative value across such a juncture for the cylindrical bodies having ogival or ellipsoidal noses studied.

Acknowledgment

The work described in this Note was supported by NASA Langley Research Center Grant NSG-1357, Mr. Neal T. Frink, technical monitor.

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Modifying TRANDES to Obtain Given Lift Coefficient

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Nomenclature

C_l	= section lift coefficient
C_p	= pressure coefficient, $(p-p_\infty)/q_\infty$
p	= local pressure, psf
P_∞	= freestream static pressure, psf
q_∞	= freestream dynamic pressure, psf
Re	= Reynolds number
x	= airfoil chordwise location, nondimensional in chord length
α	= airfoil angle of attack, deg
α_{in}	= airfoil angle of attack; refer to a normal TRANDES run using α_{out} as input
α_{out}	= airfoil angle of attack; calculated by TRANDES with convergence to C_l specified

Introduction

LOCAL flowfields in the vicinity of lifting surfaces, particularly propeller/rotary wing surfaces, can be so complex that an accurate value of angle of attack is difficult to obtain, whereas a desired lift coefficient generally can be specified rather easily.

It has been shown that the transonic airfoil analysis and design program, TRANDES,[1] does not correlate pressure well with experimental data using angle of attack as a parameter, but does provide good correlation if lift coefficient is used as the correlation parameter.[2] If one wishes to use TRANDES as a design tool, this ambiguity in angle of attack can lead to erroneous results; or it can result in an excessive number of computational runs to obtain the desired result. However, if lift coefficient can be specified and a resulting pressure distribution obtained, then this distribution should be reasonably close to the expected one.

A method[3] similar to TRANDES has been used successfully as a design tool for several years; however, this method does correlate pressure with experimental results using either angle of attack or lift coefficient as a correlation parameter. The method of Ref. 3 has only a simple turbulent boundary layer, whereas TRANDES has independent upper and lower surface boundary layers with laminar/turbulent runs and natural transition[4] as well as a low-speed maximum lift coefficient prediction method.[4] TRANDES has an improved and expanded methodology compared to the method of Ref. 3.

In keeping with the improvements to TRANDES, it would be an enhancement to provide the capability of obtaining a pressure distribution directly for a desired lift coefficient. This paper reports alterations to the program that permit convergence to lift coefficient. Details of the changes to the changes to the program may be obtained by contacting the author.

Technical Approach

TRANDES has as an independent parameter, the angle of attack α. In Ref. 1 α is an input parameter and held constant. In order to alter the method to accept a required lift coefficient, angle of attack is allowed to vary with each relaxation sweep through the grid. As angle of attack is updated with

Received Oct. 3, 1983; revision received Nov. 15, 1983

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